Logic Review Key

1a. $q \Rightarrow p$

1b. $p \Rightarrow q$

1c. Not equivalent; a kite or an isosceles trapezium (for example) can have diagonals that are equal in length.

Notes: Accept a valid sketch as reasoning.

1d. Inverse

2a.

r	p	q	$r \wedge p$	$\neg q$	$(r \wedge p) \vee \neg q$	$\neg((r \wedge p) \vee \neg q)$	$\neg(r \wedge p)$	$\neg(r \wedge p) \wedge q$
T	T	T	**T**	F	**T**	**F**	F	**F**
T	T	F	**T**	T	**T**	**F**	F	**F**
T	F	T	**F**	F	**F**	**T**	T	**T**
T	F	F	**F**	T	**T**	**F**	T	**F**
F	T	T	**F**	F	**F**	**T**	T	**T**
F	T	F	**F**	T	**T**	**F**	T	**F**
F	F	T	**F**	F	**F**	**T**	T	**T**
F	F	F	**F**	T	**T**	**F**	T	**F**

2b. tautology ***because*** columns $\neg((r \wedge p) \vee \neg q))$ and $\neg(r \wedge p) \wedge q$ are identical

OR

Tautology because there additional column representing $\neg((r \wedge p) \vee \neg q)) \Leftrightarrow \neg(r \wedge p) \wedge q$ that is all true.

3a. $(p \wedge q) \Rightarrow r$

3b.

p	q	r	$(p \wedge q)$	$(p \wedge q) \Rightarrow r$
T	T	T	**T**	**T**
T	T	F	**T**	**F**
T	F	T	**F**	**T**
T	F	F	**F**	**T**
F	T	T	**F**	**T**
F	T	F	**F**	**T**
F	F	T	**F**	**T**
F	F	F	**F**	**T**

3c. The argument is not valid since not all entries in the final column are T.

3d. (i) $\neg(p \wedge q) \Rightarrow \neg r$ **OR** $(\neg p \vee \neg q) \Rightarrow \neg r$

(ii) if it is **not the case** that the land has been purchased **and** the building permit has been obtained then the land can **not** be used for residential purposes.

OR

if (either) the land has **not** been purchased **or** the building permit has **not** been obtained then the land can **not** be used for residential purposes.

4a. If I do not have a bowl of soup then I have an ice cream.

4b.

p	q	$\neg p$	$\neg p \Rightarrow q$
T	T	**F**	**T**
T	F	**F**	**T**
F	T	**T**	**T**
F	F	**T**	**F**

4c. $q \Rightarrow \neg p$

5a. If I do not choose history then I choose psychology or I choose art.

5b.

a	p	$\neg a$	$\neg a \Rightarrow p$
T	T	F	**T**
T	F	F	**T**
F	T	T	**T**
F	F	T	**F**

5c. Neither, because not all the entries in the last column are the same.

6a. If the sun is shining, then I will go swimming.

6b. Either the sun is not shining or I will go swimming.

6c.

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	**F**	T
T	F	F	**F**	F
F	T	T	**T**	T
F	F	T	**T**	T

6d. They are (logically) equivalent.

7a. If ABCD is a square then ABCD has four equal sides.

7b. If ABCD is not a square then ABCD does not have four equal sides.

7c. Not valid because ABCD may have equal sides but will not necessarily be a square (it may be a rhombus).