

1. Some of the customers in each café were given survey forms to complete to find out if they were satisfied with the standard of service they received.

	Pete's Eats	Alan's Diner	Sarah's Snackbar	Total
Dissatisfied	16	8	16	40
Satisfied	26	20	34	80
Total	42	28	50	120

$u = \{120\}$

One of the survey forms was chosen at random. find the probability that

- (a) the form showed "Dissatisfied"; $\frac{16+8+16}{120} = \frac{40}{120} = \frac{1}{3}$
- (b) the form showed "Satisfied" and was completed at Sarah's Snackbar; $\frac{34}{120} \div 2 = \frac{17}{60}$
- (c) the form showed "Dissatisfied", given that it was completed at Alan's Diner. $\frac{8}{28}$

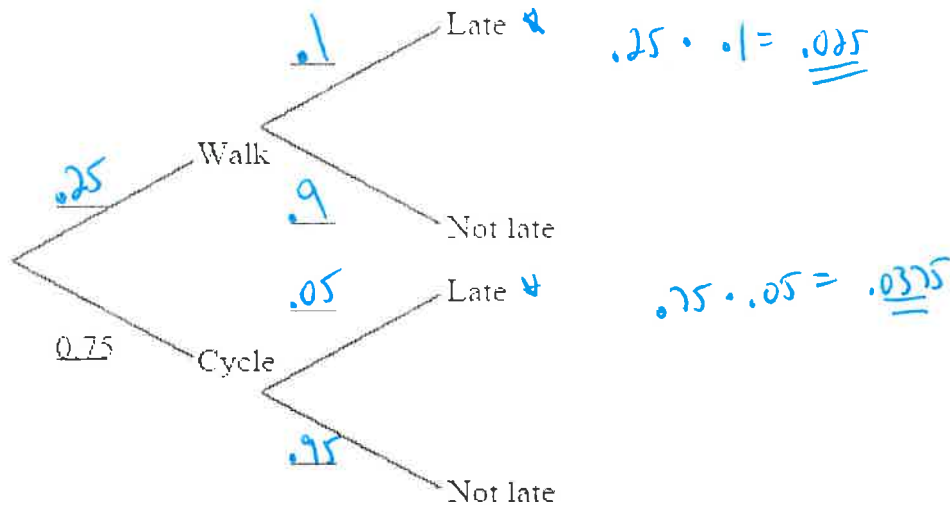
$P(A|B) = \frac{P(A \cap B)}{P(B)}$

2. Maria travels to school either by walking or by bicycle. The probability she cycles to school is 0.75.

If she walks, the probability that she is late for school is 0.1.

If she cycles, the probability that she is late for school is 0.05.

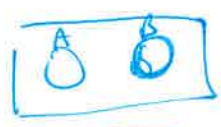
(a) Complete the tree diagram below, showing the appropriate probabilities.



(b) Find the probability that Maria is late for school.

$0.025 + 0.0375 = 0.0625 \approx 6.25\%$

3. For events A and B , the probabilities are $P(A) = \frac{4}{13}$ and $P(B) = \frac{5}{13}$.



(a) If events A and B are mutually exclusive, write down the value of $P(A \cap B)$.

$P(A \cap B) = \emptyset$

(b) If events A and B are independent, find the value of $P(A \cap B)$.

$P(A \cap B) = P(A)P(B) = \left(\frac{4}{13}\right)\left(\frac{5}{13}\right) = \frac{20}{169} \approx 11.8\%$

(c) If $P(A \cup B) = \frac{7}{13}$, find the value of $P(A \cap B)$.

$\frac{7}{13} = \frac{4}{13} + \frac{5}{13} - P(A \cap B)$ so $P(A \cap B) = \frac{2}{13} \approx 15.4\%$

4. A weighted die has 2 red faces, 3 green faces and 1 black face. When the die is thrown, the black face is three times as likely to appear on top as one of the other five faces. The other five faces have equal probability of appearing on top.

The following table gives the probabilities.

Red 1	Red 2	Green 1	Green 2	Green 3	Black
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{m}{8}$	$\frac{1}{8}$	$\frac{n}{8}$

$= \frac{8}{8}$

(a) Find the value of

(i) $m = 1$

(ii) $n = 3$

The die is thrown once.

$\left(\frac{1}{2}\right)$

(b) Given that the face on top is not red, find the probability that it is black.

$P(B) = \frac{3}{8}$
 $P(B|R') = \frac{P(B \cap R')}{P(R')}$

The die is now thrown twice.

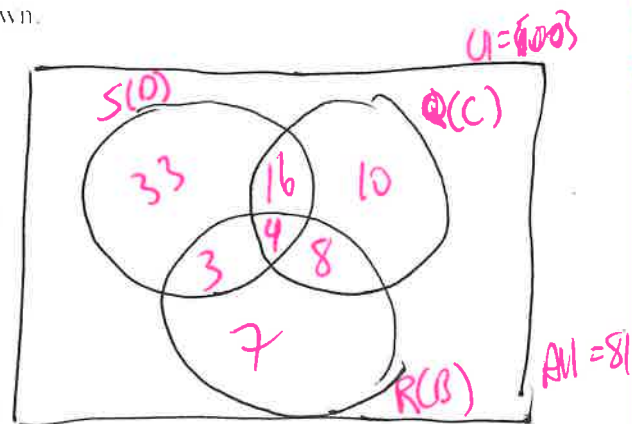
$P(B \text{ then } B) = \frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64} \approx 14.1\%$

$P(R) = \frac{4}{8}$ $P(R') = \frac{6}{8}$
 $\frac{3}{8} \div \frac{6}{8} = \frac{1}{2}$

(c) Calculate the probability that black appears on top both times.

5. A survey of 100 families was carried out, asking about the pets they own. The results are given below.

- 56 owned dogs (S)
- 38 owned cats (Q)
- 22 owned birds (R)
- 16 owned dogs and cats, but not birds
- 8 owned birds and cats, but not dogs
- 3 owned dogs and birds, but not cats
- 4 owned all three types of pets



(a) Draw a Venn diagram to represent this information.

(b) Find the number of families who own no pets. $100 - 81 = 19$

(c) Find the percentage of families that own exactly one pet. $\frac{33 + 10 + 7}{100} = \frac{1}{2}$

(d) A family is chosen at random. Find the probability that they own a cat, given that they own a bird.

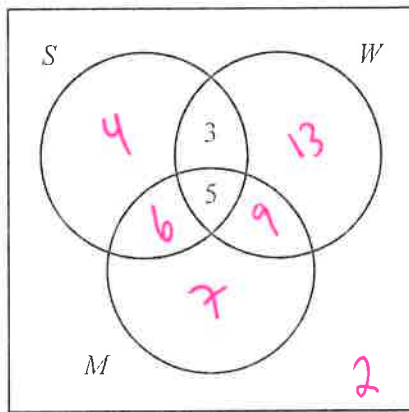
$P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{12}{22} \approx 54.6\%$

6. There are 49 mice in a pet shop.
 30 mice are white. $30 - 17$
 - 27 mice are male. $27 - 18$
 - 18 mice have short tails. $18 - 14$
 - 8 mice are white and have short tails.
 - 11 mice are male and have short tails. $6 + 5$
 - 7 mice are male but neither white nor short-tailed.
 - 5 mice have all three characteristics and
 - 2 have none.

Work up!

Copy the diagram below to your examination script.

$$U = \{49\}$$



U

W represents white mice.
 M represents male mice.
 S represents short-tailed mice.

- (a) Complete the diagram, using the information given in the question.
 (b) Find (i) $n(M \cap W) = 14$
 (ii) $n(M' \cup S) = 22 + 11 = 33$ or $15 + 18 = 33$

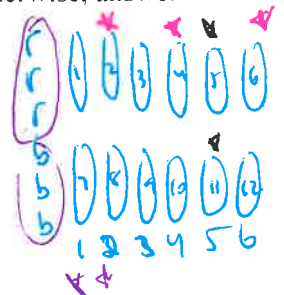
Two mice are chosen without replacement.

- (c) Find P (both mice are short-tailed).

$$\frac{18}{49} \cdot \frac{17}{48} = \frac{306}{2352} \approx 13\%$$

7. Heinrich rolls two 6-sided dice at the same time. One die has three red sides and three black sides. The other die has the sides numbered from 1 to 6. By means of a tree diagram, table of outcomes or otherwise, answer each of the following questions.

- (a) How many different possible combinations can he roll? 12
 (b) What is the probability that he will roll a red and an even number? $\frac{3}{12} \approx 25\%$
 (c) What is the probability that he will roll a red or black and a 5? $\frac{2}{12} \approx 16.7\%$
 (d) What is the probability that he will roll a number less than 3? $\frac{4}{12} \approx 33.3\%$



8. Note: For this question, it is important that you show your working and explain your method clearly.

A box contains 10 coloured light bulbs, 5 green, 3 red and 2 yellow. One light bulb is selected at random and put into the light fitting of room A.

$$U = \{10\}$$

- (a) What is the probability that the light bulb selected is
 (i) green? $P(G) = \frac{5}{10} \approx \frac{1}{2}$ or 50%
 (ii) not green? $P(G) + P(G)' = 1$ so $P(G)' = \frac{1}{2}$

A second light bulb is selected at random and put into the light fitting in room B.

(b) What is the probability that

(i) the second light bulb is green given the first light bulb was green?

$$P(G|G) = \frac{4}{9}$$

(ii) both light bulbs are not green?

$$P(G' \text{ then } G') = \frac{5}{10} \cdot \frac{4}{9} = \frac{20}{90} \approx 22.2\%$$

(iii) one room has a green light bulb and the other room does not have a green light bulb?

A third light bulb is selected at random and put in the light fitting of room C.

$P(G \text{ and } G')$

(c) What is the probability that

(i) all three rooms have green light bulbs?

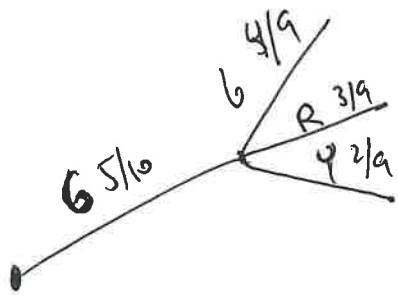
$$P(G) \text{ then } P(G) \text{ then } P(G) = \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} = \frac{60}{720} \approx 8.33\%$$

(ii) only one room has a green light bulb?

$$41.7\%$$

(iii) at least one room has a green light bulb?

$$91.7\%$$



$$G \text{ and } G' = \frac{5}{10} \cdot \frac{5}{9} = \frac{25}{90}$$

$$\frac{50}{90} = \frac{5}{9} \approx 55.6\%$$



$$G' \text{ and } G = \frac{5}{10} \cdot \frac{5}{9} = \frac{25}{90}$$

$$P(\text{only one Green}) = 3 \cdot \frac{5}{10} \cdot \frac{5}{9} \cdot \frac{4}{8} = \frac{300}{720} \approx .417 \approx 41.7\%$$

$$\begin{aligned} P(\text{at least one green}) &= 1 - P(\text{no green}) \\ &= 1 - (5/10 \cdot 4/9 \cdot 3/8) \\ &= 1 - \frac{60}{720} \\ &= \frac{11}{12} \approx 91.7\% \end{aligned}$$